



*Contract to purchase farm products is a means of reducing risks caused by price fluctuations. If the contract is not carried out, loss is much bigger than compensations for both parties because buying party does not have raw materials it needs while the selling one cannot sell its produce. One of the main causes of breach of contract is the inappropriate and unscientific definition of constraint (or fine). This research tries to identify coefficient of constraints using the game theory and find that the coefficient of constraint on implementation of the contract to buy farm product should be equal to or greater than  $\frac{P^U - P^D}{P^F}$  % of the value of the contract.*

Keywords: Nash equilibrium; normal-form game; price at the end of period; constraint.



One of measures to reduce risk caused by fluctuations in price of farm products is the contract to sell between peasants and companies (food processing or agricultural materials companies, or exporters of farm products) that sets some fine or compensation as a measure to ensure interests of involved parties. This model, however, is usually broken by fluctuations in market price at harvest time and inappropriate compensation, because the coefficient of constraint on implementation of the contract is not determined scientifically. This paper has two main contents: (1) Scientific basis for identification of coefficient of constraint; and (2) policy implications that aim at perfecting the contract to sell with a view to reducing risks for peasants.

## I. SCIENTIFIC BASIS FOR ESTABLISHMENT OF COEFFICIENT OF CONSTRAINT

### 1. Basic arguments of the game theory

According to Mankiw, 2003 [1], the classic problem widely used as an example of the game theory is the prisoner's dilemma that could be presented in the following matrix:

**Table 1: Prisoner's dilemma**

Player	Strategy	Prisoner 2	
		<i>Staying silent</i>	<i>Betraying</i>
Prisoner 1	<i>Staying silent</i>	1,1	20,0
	<i>Betraying</i>	0,2	8,8

In the game, each player has two optional strategies: betraying or staying silent. When a pair of specific strategies is chosen, their payoffs are provided in corresponding row or column of the matrix. Thus, when Prisoner 1 decides to stay silent while Prisoner 2 choose betraying, payoff for Prisoner 1 is 20 (years in jail) and payoff for Prisoner 2 is 0 (he is released right away).

Thus, the general form of the game comprises payers, strategies available for each player and payoff received from each combination of strategies they choose.

The normal-form representation of a game is a specification of players' strategy spaces  $S_1, S_2, \dots, S_n$  and their payoff functions  $u_1, u_2, \dots, u_n$ . Symbol of the game is  $G = \{S_1, S_2, \dots, S_n; u_1, u_2, \dots, u_n\}$ .

In their dilemma, sensible payers will not select strategies that damage their payoffs. These are called dominated strategies.

In the normal-form game  $G = \{S_1, S_2, \dots, S_n; u_1, u_2, \dots, u_n\}$ , call  $s_i'$  and  $s_i''$  feasible strategies for Player  $i$  (that is,  $s_i'$  and  $s_i''$  are elements of  $S_i$ ). Strategy  $s_i'$  will be strictly dominated by strategy  $s_i''$  if the combination of feasible strategies of other payers makes the payoff from  $s_i'$  smaller than the one received from strategy  $s_i''$ :

$$u_i(s_1, s_2, \dots, s_{i-1}, s_i', s_{i+1}, \dots, s_n) < u_i(s_1, s_2, \dots, s_{i-1}, s_i'', s_{i+1}, \dots, s_n)$$

For all strategies  $(s_1, s_2, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$  they could be developed from strategy spaces  $(S_1, S_2, \dots, S_{i-1}, S_{i+1}, \dots, S_n)$  of other players.

Elimination of dominated strategies through moves with a view to perfecting the selected strategy is called iterated elimination of dominated strategies (IEDS).

In a normal-form game  $G = \{S_1, S_2, \dots, S_n; u_1, u_2, \dots, u_n\}$ , strategies  $(s_1^*, s_2^*, \dots, s_i^*, \dots, s_n^*)$  is a Nash equilibrium (Nash, 1950) [2]. If  $s_i^*$  is the best action to Player  $i$  to deal with given strategies of the remaining  $(n-1)$  players then  $(s_1^*, s_2^*, \dots, s_i^*,$

$s_{i+1}^*, \dots, s_n^*)$ :  
 $u_i(s_1^*, s_2^*, \dots, s_i^*, \dots, s_n^*) \geq u_i(s_1^*, s_2^*, \dots, s_i, s_{i+1}^*, \dots, s_n^*)$  to all feasible strategies  $s_i \in S_i$ ; that is,  $s_i^*$  is solution to the problem: Max  $u_i(s_1^*, s_2^*, \dots, s_i, s_{i+1}^*, \dots, s_n^*)$  with  $s_i \in S_i$

## 2. Hypotheses of application of the game theory

Based on the game theory, we can start a game with two players:

- $N_1$ : (peasant or producer)
- $N_2$ : (buyer or consumer)

Suppose that they have an agreement according to which  $N_1$  produce and sell goods to  $N_2$  as raw materials. The game here focuses on profits and losses generated by fluctuations in market prices of the goods in contract. Suppose that the purchasing price is a fixed one agreed upon by both parties.

Strategy spaces of the two players are  $S_1(s_1, s_2)$ , and  $S_2(s_1, s_2)$ , in which  $s_1$  is implementation of contract while  $s_2$  is breach of contract.

We have:

$S_{11}$  is strategy taken by  $N_1$  who decides to follow strategy  $s_1$ .

$S_{12}$  is strategy taken by  $N_1$  who decides to follow strategy  $s_2$ .

$S_{21}$  is strategy taken by  $N_2$  who decides to follow strategy  $s_1$ .

$S_{22}$  is strategy taken by  $N_2$  who decides to follow strategy  $s_2$ .

Suppose that payoff is  $U_1$  for  $N_1$  and  $U_2$  for  $N_2$ .

-  $U_{1ij}$  is payoff for  $N_1$  when  $N_1$  takes strategy  $i$  and  $N_2$  takes  $j$ . ( $i, j = 1, 2$ ).

-  $U_{2ij}$  is payoff for  $N_2$  when  $N_2$  take strategy  $j$  and  $N_1$  takes  $i$ . ( $i, j = 1, 2$ ).

Thus, the game is presented in the following matrix:

**Table 2: General model of application of game theory**

Player	Strategy	Player N2 (Purchasing company)	
		$S_{21}$	$S_{22}$
Player N1 (Peasant)	$S_{11}$	$U_{111}; U_{211}$	$U_{112}; U_{212}$
	$S_{12}$	$U_{121}; U_{221}$	$U_{122}; U_{222}$

Players will consider strategies taken by the

other and possible payoff to make a decision on his/her move.

## II. GAME THEORY AND SIGNATURE OF CONTRACT

Suppose that  $N_2$  agrees to buy all produce supplied by  $N_1$  after harvest at the fixed price agreed-upon by both parties. When the harvest time comes, the contract comes into effect, market price may change in the following directions:

- (1) It gets higher than the agreed price.
- (2) It gets lower than the agreed price.
- (3) It is not determinable.

### 1. The price at the end of period is higher than the agreed one

$$\text{Call } n(n = \frac{P^U - P^P}{P^F} \cdot 100\%)$$

difference between the fixed price stated in the contract ( $P^F$ ) and the market price at the end of period ( $P^U$ ). The compensation set by the contract (constraint) for breach of contract is  $m$  as a percentage of the value of the contract and,  $m \geq 0$ .

When the price at the end of period is higher, Player  $N_2$  for some reason, could not breach the contract or carry out strategy  $S_{22}$ . If so, it has to buy the farm product from the market at a high price and pay compensation for breach of contract. The game now comes from the following model:

**Table 3: Model of the game when the price at the end of period is higher**

Player	Strategy	Player N2 (Purchasing company)
		$S_{21}$
Player N1 (Peasant)	$S_{11}$	$U_{111} = - (P^U - P^F) = - [(nP^F + P^F) - P^F] = - nP^F$
		$U_{211} = (P^U - P^F) = [(nP^F + P^F) - P^F] = nP^F$
	$S_{12}$	$U_{121} = (P^U - P^F) - mP^F = [(nP^F + P^F) - P^F] - mP^F = nP^F - mP^F$
		$U_{221} = - (P^U - P^F) + mP^F = - [(nP^F + P^F) - P^F] + mP^F = mP^F - nP^F$

Source: Authors' calculations

In this case, Player  $N_1$  will follow the mixed strategy  $S_{11}$ ,  $S_{21}$  because he/she does not know how big the profit (when selling the produce at market price) or compensation for breach of contract is; while Player  $N_2$  only follows strategy  $S_{12}$ .

Call:

-  $P_{S11}$  probability that Player 1 follows strategy 1,

-  $P_{S12}$  probability that Player 1 follows strategy 2,

and

-  $P_{S21}$  probability that Player 2 follows strategy 1.

Objective function of maximizing expected interest of the two players is as follows:

Player I:

$$\text{Max: } P_{S21}[P_{S11}(-nP^F)] + P_{S21}[P_{S12}(nP^F - mP^F)] \quad (1)$$

$$\text{constraint: } P_{S21} = 1; P_{S11} + P_{S12} = 1; P_{S11} \geq 0; P_{S21} \geq 0$$

Player II:

$$\text{Max: } P_{S21}[P_{S11}(nP^F)] + P_{S21}(P_{S12}[-nP^F + mP^F]) \quad (2)$$

$$\text{constraint: } P_{S21} = 1; P_{S11} + P_{S12} = 1; P_{S11} \geq 0; P_{S12} \geq 0$$

Thus, to ensure maximum expected interest of the two players means sharing both interest and loss between them. In other words, (1) should be equal to (2) to maximize expected interest of both players.

$$P_{S21}[P_{S11}(-nP^F)] + P_{S21}[P_{S12}(nP^F - mP^F)]$$

$$= P_{S21}[P_{S11}(nP^F)] + P_{S21}(P_{S12}[-nP^F + mP^F]) \quad (3)$$

Solving (3), we have:

$$P_{S11}(-2nP^F) = P_{S12}(-2nP^F + 2mP^F)$$

$$\text{or } m = n \left(1 - \frac{P_{S11}}{P_{S12}}\right) \text{ and } \frac{P_{S11}}{P_{S12}} = \frac{n - m}{n} = a$$

with  $n > 0$ ,  $m > 0$

We now consider value  $a$ :

- $a > 0$  or  $n > m$ : Profit from breach of contract is greater than loss caused by this act. In this case, Player  $N_1$  is ready to breach the contract to sell produce to other buyers at a higher price.

- $a = 0$  or  $n = m$ : Profit from breach of contract is equal to loss caused by this act. In this case, Player  $N_1$  may choose any strategy from two strategies  $S_{11}$  and  $S_{12}$  to carry out his/her move.

- $a < 0$  or  $n < m$ : Profit from breach of contract is smaller than loss caused by this act. In this case, Player  $N_1$  cannot take  $S_{12}$  (breaching the contract) and follow  $S_{11}$  instead.

Thus, to ensure balanced interest for both parties and proper implementation of the contract when the price at the end of period is higher, con-

straint set by the contract should be greater or equal to increase (as %) in the market price in comparison with the agreed price, or  $m \geq n$ .

## 2. The price at the end of period is lower than the agreed one

Like the above –mentioned case, a low price at the end of period makes Player N1 refuse to breach the contract because this act causes a double loss for N<sub>1</sub> (low selling price and a compensation for N<sub>2</sub>).

Suppose that  $k$  ( $k = \frac{P^F - P^D}{P^F} > 0$ ) is the difference between price at the end of period ( $P^D$ ) and the agreed price ( $P^F$ ). Coefficient of constraint on implementation of contract is  $z$  (as % of the value of contract;  $z \geq 0$ ). The problem is as follows:

$$P_{S11} = 1; P_{S21} + P_{S22} = 1; P_{S21} \geq 0; P_{S22} \geq 0$$

Solving the two problems, we have:

$$z = k(1 - \frac{P_{S21}}{P_{S22}}) \text{ and } \frac{P_{S21}}{P_{S22}} = \frac{(k - z)}{k} = b$$

We now consider value  $b$ :

- $b > 0$  or  $k > z$ : Player N<sub>2</sub> will breach the contract (by refusing to buy produce of peasant at agreed price to buy farm product from the market at a lower price).

- $b = 0$  or  $k = z$ : Player N<sub>2</sub> may carry out or breach the contract because interest in both moves is similar.

- $b < 0$  or  $k < z$ : Player N<sub>2</sub> never breaches the contract because it may causes a loss equal to  $(z - k)$ .

**Table 4: Model of the game when the price at the end of period is lower**

Player	Strategy	Player N2 (Purchasing company)	
		S <sub>21</sub>	S <sub>22</sub>
Player N1 (Peasant)	S <sub>11</sub>	$U_{111} = (P^F - P^D) = P^F - (P^F - kP^F) = kP^F$	$U_{112} = -(P^F - P^D) + zP^F = -[P^F - (P^F - kP^F)] + zP^F = zP^F - kP^F$
		$U_{211} = -(P^F - P^D) = -[P^F - (P^F - kP^F)] = -kP^F$	$U_{222} = (P^F - P^D) + zP^F = [P^F - (P^F - kP^F)] - zP^F = kP^F - zP^F$

Source: Authors' calculations

Player N<sub>2</sub> will work out a mixed strategy based on optional S<sub>21</sub> and S<sub>22</sub>

Thus, problem of maximization of expected interest of the two payers is as follows:

Player I:

$$\text{Max: } P_{S11}[P_{S21}(kP^F)] + P_{S11}[P_{S22}(-kP^F + zP^F)]$$

constraint:

$$P_{S11} = 1; P_{S21} + P_{S22} = 1; P_{S21} \geq 0; P_{S22} \geq 0$$

Player II:

## 3. When the price at the end of period is not determinable

In this case, the two players do not know whether the price at the end of period is lower or higher, and therefore, they cannot guess the next move of their partner.

Call  $t$  the coefficient of constraint, model of the problem with a coefficient of constraint in this case is as follows:

**Table 5: Model of the game when the price at the end of period is not determinable**

Player	Strategy	Player N2 (Purchasing company)	
		S <sub>21</sub>	S <sub>22</sub>
Player N1 (Peasant)	S <sub>11</sub>	$U_{111} = -nP^F + kP^F$	$U_{112} = nP^F - kP^F + tP^F$
		$U_{211} = nP^F - kP^F$	$U_{212} = -nP^F + kP^F - tP^F$
	S <sub>12</sub>	$U_{121} = nP^F - kP^F - tP^F$	$U_{122} = nP^F - kP^F$
		$U_{221} = -nP^F + kP^F + tP^F$	$U_{222} = -nP^F + kP^F$

Source: Authors' calculations

$$\text{Max: } P_{S11}[P_{S21}(-kP^F)] + P_{S11}[P_{S22}(kP^F - zP^F)]$$

constraint:

Thus, the problem of maximization of expected interest for two players is as follows:



Player I:

Max:  $P_{S21}[P_{S11}(-nP^F + kP^F)] + P_{S21}[P_{S12}(nP^F - kP^F - tP^F)] + P_{S22}[P_{S11}(nP^F - kP^F + tP^F)] + P_{S22}[P_{S12}(nP^F - kP^F)]$   
 constraint:  $P_{S11} + P_{S12} = 1$ ;  $P_{S21} + P_{S22} = 1$ ;  $0 \leq P_{S11}$ ,  $P_{S12} \leq 1$

Player II:

Max:  $P_{S11}[P_{S21}(nP^F - kP^F)] + P_{S11}[P_{S22}(-nP^F + kP^F - tP^F)] + P_{S12}[P_{S21}(-nP^F + kP^F + tP^F)] + P_{S12}[P_{S22}(-nP^F + kP^F)]$

constraint:  $P_{S11} + P_{S12} = 1$ ;  $P_{S21} + P_{S22} = 1$ ;  $0 \leq P_{S11}$ ,  $P_{S12} \leq 1$

We solve the above problem on condition that the expected interest of the two players is maximized:

$P_{S21}[P_{S11}(-nP^F + kP^F)] + P_{S21}[P_{S12}(nP^F - kP^F - tP^F)] + P_{S22}[P_{S11}(nP^F - kP^F + tP^F)] + P_{S22}[P_{S12}(nP^F - kP^F)]$   
 $= P_{S11}[P_{S21}(nP^F - kP^F)] + P_{S11}[P_{S22}(-nP^F + kP^F - tP^F)] + P_{S12}[P_{S21}(-nP^F + kP^F + tP^F)] + P_{S12}[P_{S22}(-nP^F + kP^F)]$

$$t = (n - k) \left( \frac{P_{S21}(P_{S12} - P_{S11}) + P_{S22}}{(P_{S21}P_{S12} - P_{S22}P_{S11})} \right)$$

$$= \frac{P^U - P^D}{P^F} \left( \frac{P_{S21}(P_{S12} - P_{S11}) + P_{S22}}{P_{S21}P_{S12} - P_{S22}P_{S11}} \right)$$

And the coefficient of constraint when the price at the end of period is not determinable is:

$$t = (n - k) \left( \frac{P_{S21}(P_{S12} - P_{S11}) + P_{S22}}{P_{S21}P_{S12} - P_{S22}P_{S11}} \right)$$

with  $(n - k) = \frac{P^U - P^D}{P^F}$  as the base coefficient; and  $\frac{P_{S21}(P_{S12} - P_{S11}) + P_{S22}}{P_{S21}P_{S12} - P_{S22}P_{S11}}$  considered as coefficient of reaction to two cases of changes in price depending on probability of implementation of their strategies (carrying out or breaching the contract).

Thus, the coefficient of constraint in the contract may change reactions of involved parties. When the market price is higher than the agreed one, Player  $N_1$  will not breach the contract because profit from this act is not bigger than fine for the breach. Similarly, purchasing company will not breach the contract when the market price falls.

### III. POLICY IMPLICATIONS

**Firstly**, the coefficient of constraint could be included in the contract to sell. When the price at the end of period is determinable, results of this research could be used to identify the coefficient of constraint. The method is as follows:

- When the price at the end of period is surely

higher than the agreed one, the coefficient in the contract ( $m\%$ ) should be greater or equal to the increase, as percentage ( $n\%$ ), in the market price in comparison with the agreed price, or  $m \geq n$ .

- When the price at the end of period is surely lower than the agreed one, the coefficient in the contract ( $z\%$ ) should be greater or equal to the decrease, as percentage ( $k\%$ ), in the market price in comparison with the agreed price, or  $z \geq k$ .

- In fact, both parties cannot estimate the market price at the end of period as lower or higher than the agreed one. In this case, the coefficient of constraint in the contract, according to the research, will be

$$\frac{(P^U - P^D)}{P^F} \left( \frac{P_{S21}(P_{S12} - P_{S11}) + P_{S22}}{P_{S21}P_{S12} - P_{S22}P_{S11}} \right)$$

Prices  $P^U$  and  $P^D$  are calculated from the highest and lowest levels of prices of farm products in the past combined with predictions of reasonable trend of the market price.

- Value of

$$\frac{P_{S21}(P_{S12} - P_{S11}) + P_{S22}}{P_{S21}P_{S12} - P_{S22}P_{S11}}$$

can be considered as coefficient of reactions of the two parties to changes in market prices. The coefficient of reaction of the two parties depends on probability of implementation or breach of the contract. When the price at the end of period is certainly higher or lower than the agreed one, and suppose that the peasant will breach the contract when the price is higher and the purchasing company will do the same when the price is lower, the coefficient of reaction is always equal to 1. Thus, the coefficient of constraint is equal to the base coefficient  $\frac{P^U - P^D}{P^F}$ .

Suppose that a contract to sell corn is signed by peasants and a animal feed company. Market data in the past allow us to fix  $P^U$  at VND4,200/kg,  $P^D$  at VND2,800/kg,  $P^F$  at VND3,600/kg, then calculation of coefficient of constraint is as follows:

- When the market price is higher than the agreed one, the coefficient should be equal to or greater than

$$\frac{P^U - P^F}{P^F} = \frac{4.200 - 3.600}{3.600} \cdot 100\% = 17\%$$

of the value of contract.

- When the market price is lower than the

agreed one, the coefficient should be equal to or greater than

$$\frac{P^F - P^D}{P^F} = \frac{3.600 - 2.800}{3.600} \cdot 100\% = 22\%$$

of the value of contract.

- And when the market price is not determinable, the coefficient should be equal to or greater than

$$\frac{P^U - P^D}{P^F} = \frac{4.200 - 2.800}{3.600} \cdot 100\% = 39\%$$

of the value of contract.

The calculation shows that the coefficient of constraint when the market price is not determinable will be greater because the fine for breach of contract is heavier in comparison with other cases. This calculation allows us to suggest that the coefficient of constraint on purchase of farm products should be equal to or greater than

$$\frac{P^U - P^D}{P^F} \% \text{ of the value of contract.}$$

**Secondly**, because the coefficient of constraint may fail to anticipate other possible risks, such as loss caused by shortage of raw materials of purchasing company, decay of farm products, storage and preservation costs, or expenses on slow sale of farm products, we suggest that an intermediary between the two parties is necessary. This intermediary, with some fee, will see to it that the contract is implemented properly at the end of period. This practice is like an insurance against price fluctuations, and may minimize risk caused by such fluctuations■

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