Demonstration of Some Formulas Relating to Tax Policy Analysis

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When learning the subject of Tax Policy Analysis in the postgraduate course of Finance-Banking, Term 15, students have to face some formulas to calculate necessary quantities, for example, demand price, supply price, tax-derived loss, total tax revenues...

Nevertheless, we detect some inadequacies and limitations of these formulas, so we like to present our demonstrations for these formulas with the aim to provide supplements for the subject of tax policy analysis.

This article will demonstrate formulas to calculate tax loss, total tax revenues, demand price, supply price by means of mathematical tools including illustration figures.

I. TAX LOSS

Let:

P : Before-tax equilibrium price

Q : Equilibrium output

t : Tax rate

T : Unit tax

ΔQ : Difference between after-tax and before-tax output

Pd : After-tax demand price

Ps : After-tax supply price

ΔPd : Different between after-tax and before-tax demand prices

ΔPs : Different between after-tax and before-tax supply prices

ΔP = ΔPd + ΔPs

Ed : Price elascity of demand

Es : Price elascity of supply

1. Special cases

A. Completely elastic supply:

- Demonstration:

The value of tax loss (W) is the area of triangle ABC

W = ½*ΔP*ΔQ

And : Ed = (ΔQ/Q)/(ΔP/P) Or: ΔQ = (ΔP/P)*Q*Ed

then: W = ½*($ΔP^2$/P)*Q*Ed (1)

In addition: ΔP = T = t*P

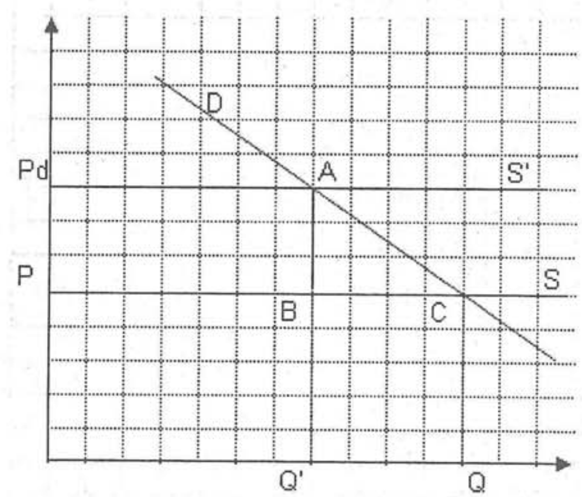
Then:W = ½*t^2*P^2/P*Q*Ed

Or : W = ½*P*Q*Ed*t^2 (2)

* Empirical comparison:

In Figure 1, we have: P = 5 ; Q = 11 ; t = 3/5 ; Ed = (4/11)/(3/5)

Figure 1: Completely elastic supply



Apply the formula: W = ½*5*11*(20/33)*$(3/5)^2$

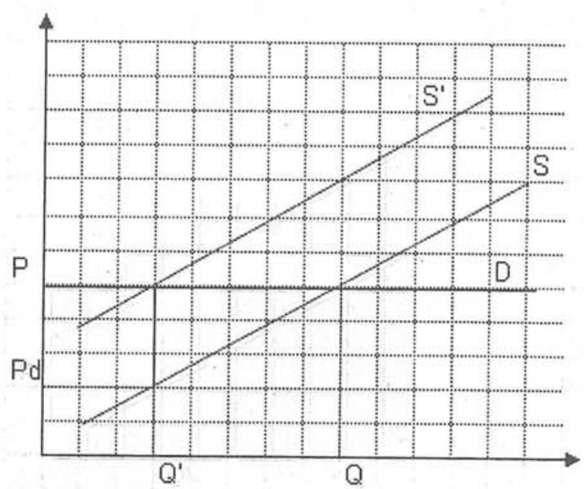
We get: W = 6

This value is suitable for the reality in Figure 1: W = ½*ΔP*ΔQ = ½*3*4 = 6

B. Completely elastic demand :

- Demonstration:

Figure 2: Completely elastic demand



According to the general formula, we have: W = ½*ΔP*ΔQ

And: Es = (ΔQ/Q)/(ΔP/P) Or: ΔQ = (ΔP/P)*Q*Es

Then: W = ½*($ΔP^2$/P)*Q*Es (3)

When the demand is completely elastic, we have:

Tax: $T = \Delta P = t*P$

Then: $W = ½*t^2*P^2/P*Q*Es$

Or: $W = ½*P*Q*Es*t^2$ (4)

- Empirical comparison :

According to Figure 2, we have: P = 5 ; Q = 8 ; t = 3/5 ; Es = (5/8)/(3/5)

Then: $W = ½*5*8*5/8*5/3*(3/5)^2$

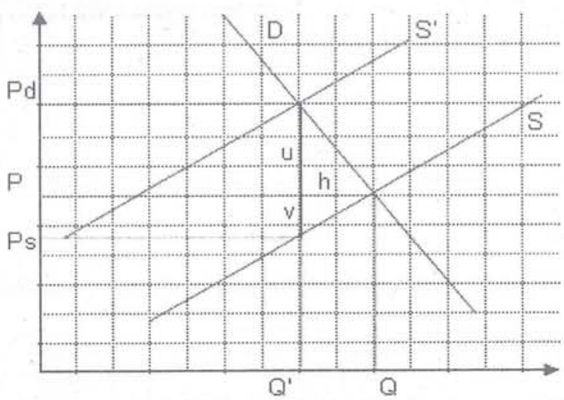
W = 7.5

This value is suitable for the figure: $W = ½*\Delta P*\Delta Q = ½*5*3 = 7.5$

1. General case :

- Demonstration:

Figure 3: The general case



In the general case, where :

$\Delta Pd = Pd - P$

$\Delta Ps = P - Ps$

Then: $Ed = (\Delta Q/Q)/(\Delta Pd/P) \Rightarrow \Delta Pd = (\Delta Q/Q)/(Ed/P)$

$Es = (\Delta Q/Q)/(\Delta Ps/P) \Rightarrow \Delta Ps = (\Delta Q/Q)/(Es/P)$

Combine these two equations, then: $\Delta Pd = \Delta Ps*Es/Ed$ (5)

In addition, we have : t = T/P =

$= (\Delta Pd + \Delta Ps)/P \Rightarrow t*P = \Delta Pd + \Delta Ps$

Replace ΔPd with the equation (5), then: $t*P = \Delta Ps*(Es/Ed) + \Delta Ps$

$t*P = \Delta Ps*[Es/Ed + 1]$

$\Delta Ps = t*P/[Es/Ed + 1]$ (6)

Apply (6) into (5) then : $\Delta Pd = t*P/(1 + Ed/Es$ (7)

The loss W is: $W = ½*T*\Delta Q = ½*t*P * \Delta Q$

Note that: $\Delta Q = (\Delta Pd/P)*Q*Ed$ (8)

$½*t*P*(\Delta Pd/P)*Q*Ed$

$½*t*Q*Ed*\Delta Pd$

Apply (7) into (8) and change, then:

$W = ½*t^2*P*Q*Ed/[1 + Ed/Es]$

Or : $W = ½* t^2*P*Q*[1/(1/Ed +1/Es)]$ (9)

In special cases :

When the supply is completely elastic (Es -> ∞), Formula (9) is easily changed into Formula (2);

When the demand is completely elastic (Ed->∞), the general formula is easily changed into Formula (4).

Empirical comparison:

In Figure 3, we have P = 6 ; Q = 9 ; t = 4.5/6 = 0.75 ; Ed = 4/9 ; Es = 8/9

We apply the above formula to calculate and get W = 4.5

This value is suitable for the reality: W = ½*4,5*2 = 4.5.

II. TOTAL TAX REVENUE

1. Demonstration by theory:

The formula to calculate total tax revenue: $TTR = T*(Q-\Delta Q)$

Apply (8) into this equation, then: $TTR = t*P*Q*[1 - (\Delta Pd/P)*Ed]$

Apply (7) into the above equation:

$TTR = t*P*Q*[1 - (t*P*Ed/(P*(1 + Ed/Es))]$

Change and reduce, then:

$TTR = t*P*Q*[1 - t/(1/Ed + 1/Es)]$ (10)

2. Demonstration by figure: *(Figure3)*

In Figure 3 we have:

P = 6 , Q = 9 , Ed = 4/9 , Es = 8/9 , t = 0.75

Apply these figures to the above equation, then:

TTR = 0.75*6*9*[1 – 0.75/(9/4 + 9/8)]

TTR = 31.5

This value is suitable for the figure:

$TTR = T*(Q - \Delta Q) = t*P*(Q - \Delta Q) = 0.75*6*(9 - 2) = 31.5$

3. Comparison with the book's formula

$TTR = t*P*Q + t^2*P*Q*Ed*(1 + Es)/[Es - Ed*(1 + t)]$

$= 0.75*6*9 + 0.75^2*6*9*(4/9)*[1 + (8/9)]/[(8/9) - (4/9)*(1 +0.75)]$

TTR = 270 (!)

This value is not suitable for the reality.

III. SUPPLY PRICE

$Ps = P - \Delta Ps$

Apply equation (6) to the above equation, then: Ps = P – t*P/(1 + Es/Ed)

Ps = P[1 – t/(1 + Es/Ed)] (11)

Practical test (Figure 3): Ps = 6*[1 – 0.75/(1 + (8/9)/(4/9))]

Ps = 4.5 (Right!)

IV. DEMAND PRICE

$Pd = P + \Delta Pd$

Apply equation (7) to the above equation, then: Pd = P + t*P*Es/(Ed + Es)

Or : Pd = P[1 + t/(1 + Ed/Es)] (12)

Practical test (Figure 3) : Pd = 6*[1 + 0.75/(1 + (4/9)/(8/9))]

Pd = 9 (Right!)■